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Simulation of Current-Induced Scour in Movable-Bed Inlet Models

by Steven A. Hughes

Purpose: This Coastal Engineering Technical Note provides guidance on using movable-bed physical models to predict erosion and deposition caused by currents in tidal inlet channels. A scaling relationship based on equilibrium scour depth allows observed model scour depths to be quantitatively scaled to full-scale dimensions. Appropriate situations are listed for which this modeling guidance can be applied.

Background: Historically movable-bed physical models of jettied and jettied inlet systems have seen limited use due to uncertainty about similitude relationships for scaling model results to full size (prototype scale). Consequently, most movable-bed inlet model results were viewed as qualitative indicators of general inlet behavior and evolution, and little attempt has been made to use movable-bed models to quantify depth of scour or amount of deposition under given flow conditions.

Area constraints in laboratory facilities generally force the prototype-to-model geometric length scale (N_L) of a typical inlet to be on the order of 75-to-1 or greater ($N_L \geq 75$). For a model in which the flow kinematics conform to the Froude similitude relationship, it is not practical to reduce the model sediment size by the geometric length scale and still have noncohesive sediment in the model. Instead, noncohesive sediment is used in the model that has a grain-size diameter relatively larger than the diameter required by Froude scaling. The consequence is that model currents need to be relatively faster than currents specified by Froude scaling to move the sediment and scour the inlet channel to a depth similar to what would occur in nature. Therefore, it is necessary to “distort” the prototype-to-model Froude velocity scale to achieve similarity in scour patterns between the model and prototype. Unfortunately, distorting the velocity scale limits the movable-bed model to situations where inlet channel scour is caused by either currents or waves, but not by both.

This note focuses on movable-bed modeling of inlet channel scour problems stemming from bedload sediment transport caused by tidal currents. Such scour is most common in the throat of inlet systems where waves have only a secondary effect because of reduced wave height or fairly deep channel depths. Situations where waves are thought to be a dominant scour mechanism are not correctly simulated by the relationship given in this note.

Scaling Relationship: An appropriate scaling relationship for movable-bed modeling of channel evolution produced by the tidal current is derived in Appendix A of this note. The scaling relation was derived by assuring that an expression for equilibrium scour depth at tidal inlets is the same in the model as in the prototype. The resulting scaling relationship is given by the expression:

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **JUN 2000**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE
Simulation of Current-Induced Scout in Movable-Bed Inlet Models (ERDC/CHL CHETN-IV-26)

5a. CONTRACT NUMBER
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Hughes, Steven A.

5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
US Army Corps of Engineers, Engineer Research and Development Center, Coastal and Hydraulics Laboratory, Vicksburg, MS

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**10**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

$$N_{\bar{v}} = \left[N_{(S_s - 1)} \right]^{1/2} N_{d_e}^{3/8} \; N_L^{1/8} \qquad (1)$$

in which the scale ratios are defined as:

$N_{\bar{v}}$ = velocity scale $[= \overline{V}_p / \overline{V}_m \; V_p / V_m]$
$N_{(Ss-1)}$ = sediment immersed specific gravity scale $[=(S_s - 1)_p/(S_s - 1)_m]$
N_{de} = sand grain size scale $[=(d_e)_p/(d_e)_m]$
N_L = geometric length scale $[=L_p/L_m]$

where the subscripts *p* and *m* represent "prototype" and "model," respectively, and the variables are defined as:

$\overline{V}$ = depth-averaged velocity
S_s = sediment specific gravity $[= \rho_s / \rho_w]$
ρ_s = mass density of sediment
ρ_w = mass density of water
d_e = sand median grain size
L = length

The $N_{(Ss-1)}$ scale ratio allows use of model sediment having different density than the prototype. However, for practical reasons quartz sand is typically used in models so $N_{(Ss-1)} = 1$, reducing Equation 1 to a simpler scaling relationship given by:

$$N_{\bar{v}} = N_{d_e}^{3/8} \; N_L^{1/8} \qquad (2)$$

The N_{de} scale ratio in Equation 2 compensates for model sediment that is relatively larger than if it had been geometrically scaled according to the length scale, N_L, as required by strict geometric scaling. Consequently, the velocity scale in Equations 1 and 2 is a "distorted" velocity scale. In other words, the sediment grains in the model are larger than they should be, so the model currents need to be faster than Froude-scaled currents to achieve the same equilibrium depth of scour. For the extremely rare case when the prototype sediment can be scaled geometrically, then $N_{de} = N_L$, and Equation 2 reduces to the Froude velocity scale, given by

$$N_{\bar{v}} = N_L^{1/2} \qquad (3)$$

Scaling Validation: The scaling relationship given by Equation 2 is validated by demonstrating that results from small-scale laboratory inlets at (or approaching) equilibrium follow the same tidal-prism relationship as real inlets when the model results are scaled using Equation 2.

O'Brien (1931, 1969) established the relationship between inlet equilibrium cross-sectional area and tidal prism. O'Brien's original work was extended and refined by Jarrett (1976). The solid line on Figure 1 is Jarrett's Area - Prism relationship for all inlets. The adjacent dashed lines are the 95 percent confidence limits. The cluster of data points in the lower left corner of Figure 1

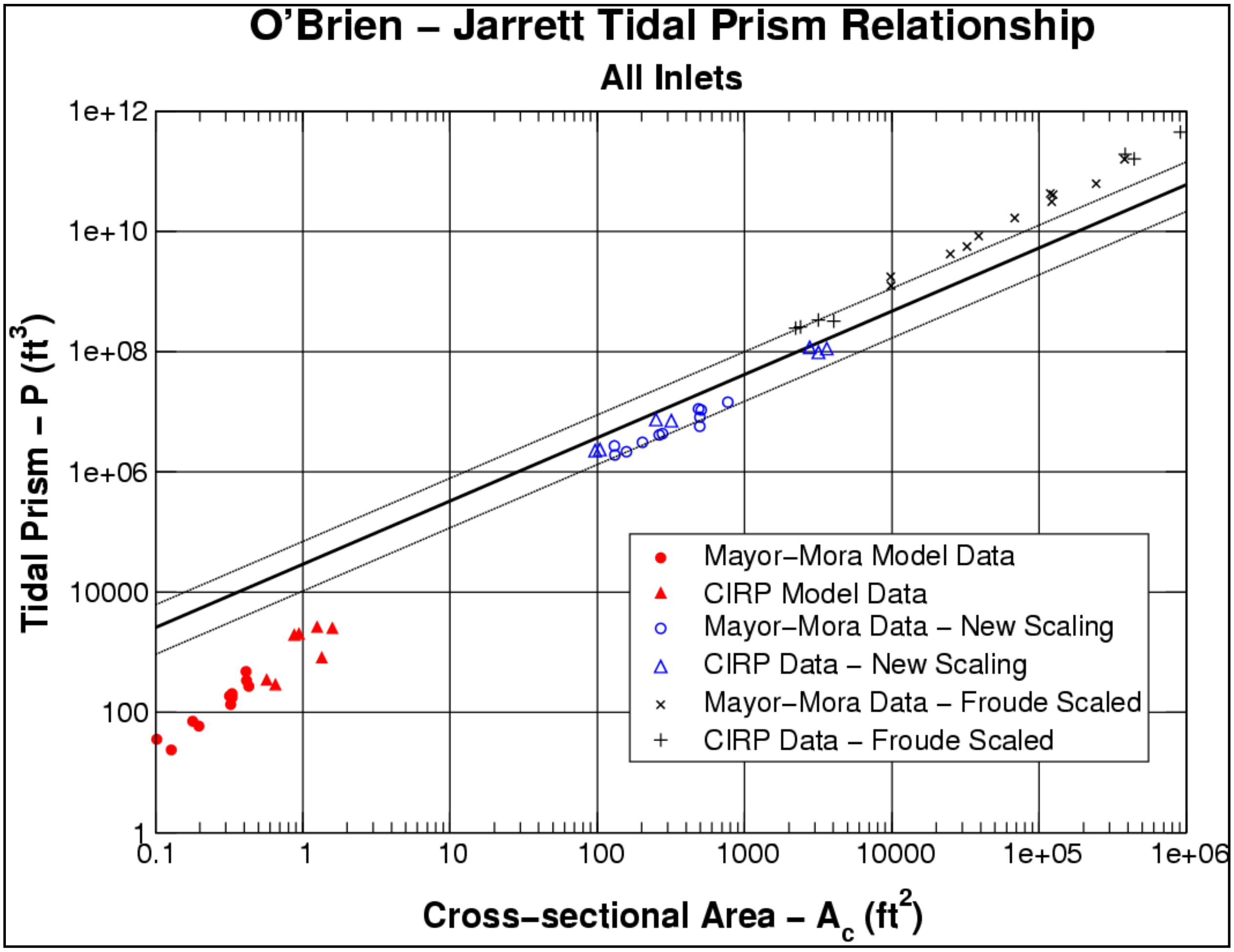

Figure 1. Model inlet results scaled to prototype (To obtain meters, multiply feet by 0.3048)

are results from 18 small-scale movable-bed model experiments. The solid circles are 11 experiments conducted by Mayor-Mora (1977), and the solid triangles are 7 experiments conducted as part of the Coastal Inlets Research Program (CIRP). These experiment results fall well below the tidal prism relationship indicating that the inlet cross-sectional area in the model was an order of magnitude larger than what might be expected for the model tidal prism.

Model results were scaled to prototype size using both conventional Froude scaling and the scaling relationship given in this technical note. For both methods the tidal prism can be represented as:

$$P = \frac{Q_{max}\, T}{\pi} = \frac{A\, V_{max}\, T}{\pi} \tag{4}$$

where

P = tidal prism
Q_{max} = maximum flow discharge
T = tidal period

A = inlet throat cross-sectional area
V_{max} = average maximum velocity at inlet throat

The tidal prism scale ratio is given by the prototype-to-model ratio of Equation 4, i.e.,:

$$N_p = N_V \; N_T \; N_A \tag{5}$$

where N_V, N_T, and N_A are the velocity scale, tidal period time scale, and cross-sectional area scale, respectively.

Froude Scaling. Noting for Froude-scaled models $N_V = N_T$, $N_A = (N_L)^2$, and $N_L = (N_T)^2$, the prism scale can be expressed in terms of just the time scale. Thus, the model results can be scaled to prototype area and prism using the Froude-scale equations:

$$A_p = A_m \left(N_T\right)^4 \quad \text{and} \quad P_p = P_m \left(N_T\right)^6 \tag{6}$$

where once again the subscripts p and m represent prototype and model, respectively. Model tidal periods were used to establish the time scale, and the corresponding length scales were then fixed by the Froude scale. (Resulting prototype length scales ranged between $50 \le N_L \le 1{,}388$.) The Froude-scaled experiment results are shown as x-symbols (Mayor-Mora 1977) and plus-signs (CIRP data) on Figure 1. All but one value are outside Jarrett's 95 percent confidence limit, with cross-sectional areas smaller then would be expected for the scaled-up tidal prism. This is logical because the model sediment diameter was much larger than required for Froude scaling, so it was not as easily scoured from the model inlet.

New Scaling Relationship. Scaling the small-scale model results to prototype using the scale relationship given in this technical note uses the same tidal prism scale given by Equation 5. But instead of Froude velocity scaling, Equation 2 is substituted for N_V in Equation 5, yielding

$$N_p = N_{d_e}^{3/8} \; N_T \; N_L^{17/8} \tag{7}$$

The relationship between the length and time scales is found from the identity $N_V = N_L/N_T$ or:

$$N_L = N_{d_e}^{3/7} \; N_T^{8/7} \tag{8}$$

where N_V is given by Equation 2. Substituting Equation 8 into Equation 7 results in the following relationships for scaling area and prism to prototype scale:

$$A_p = A_m \left(N_L\right)^2 = A_m \left(N_{d_e}\right)^{6/7} \left(N_T\right)^{16/7} \quad \text{and} \quad P_p = P_m \left(N_{d_e}\right)^{9/7} \left(N_T\right)^{24/7} \tag{9}$$

The model sediment grain size was 0.34 mm in Mayor-Mora's (1977) experiments, and the CIRP experiments used grain sizes of 0.13 mm and 0.26 mm. Because 0.34 mm is typical for real tidal inlets, this value was chosen as the prototype grain size. Therefore, N_{de} was unity for Mayor-

Mora's data and either 1.3 or 2.6 for the CIRP data. Using the same time scales as before, CIRP and Mayor-Mora's (1977) model results were scaled to prototype and plotted as hollow circles and triangles in Figure 1. (Resulting prototype length scales ranged between $10 \leq N_L \leq 70$.) Although not perfect, most points fall within the 95 percent confidence band associated with the tidal-prism relationship, indicating reasonable agreement with real inlets. Also note that sediment grain size was included in the scaling, although its influence is relatively minor.

Appropriate Movable-Bed Modeling Applications: The most critical aspect of any modeling technology is understanding which real-world situations are appropriate candidates for modeling. This is particularly true for movable-bed physical models because of potential "scale effects" related to the model sediment.

The scaling relationship given in this note reduces sediment scale effects by attempting to assure that the balance between the boundary layer shear stress acting on the bottom and the critical shear stress of the bed material is preserved in the model. This "shear stress balance" restricts the modeling technology to portions of real inlets with the following characteristics:

- Bottom scour is primarily due to the tidal current
- Concurrent wave action is small and does not contribute significantly to sediment transport
- Sediment is transported in bed-load mode (approaching equilibrium)
- Sediment is noncohesive with only minor cohesive components

Typical inlet regions and processes that could be modeled using the scaling guidance presented in this technical note are listed as follows:

> **Inlet Throat Section.** All or portions of a structured or unstructured inlet throat section could be modeled. However, attempting to model entire throat sections for large inlets is not practical without introducing geometric distortion. (Geometric distortion is possible using the proposed scaling guidance, but several other factors must be considered as well.)
>
> **Effects of Flow Jets.** Jet flow features created by inlet jetty planform geometry will be reproduced by the model, and the model bed evolution will respond accordingly. However, the relatively faster model velocity of the jet will enhance flow entrainment at the jet boundary, and this effect should be analyzed.
>
> **Effects of Structures.** Localized bed evolution adjacent to the channel side of jetty structures will be correctly simulated, as will scour at the tip of training structures. Scour at free-standing bridge piers will probably not be in similitude because the dominant *horseshoe vortex* causing scour at vertical piers and piles is a different mechanism than the shear-stress balance assumed in this scaling relationship derivation.

Inlet Modifications. Effects to channel depth brought about by jetty modification and extension, construction of additional structures, and navigation channel realignment can be simulated. However, regions of deposition will require a source of infilling sediment.

Modeling Caveats: Two caveats apply to movable-bed modeling of the situations described above. First, this modeling technology pertains only to the equilibrium condition where the movable bed no longer evolves. Sediment transport rates and the time required to reach equilibrium are not correctly simulated in the movable-bed model. In other words, the model will provide reasonable predictions of the final equilibrium bed configuration, but no information is gained regarding how long this evolution will take in the real world. This is actually beneficial because it precludes having to simulate tidal cycles in the model. At real inlets, the final equilibrium condition is maintained for the most part by the peak ebb or flood flow. During the rest of the tide stage, currents are reduced and sediment motion is greatly reduced. Therefore, in the movable-bed model we only need to simulate the maximum flow condition long enough to achieve an equilibrium.

The second caution applies to deposition areas in the movable-bed model. Sediment deposition will occur in areas where the local flow velocity falls below the sediment incipient motion threshold. Sediment moved into this region is deposited. The scaling premise of the movable-bed model scaling relationship covers this situation, but there must be available sediment to have deposition. At real inlets, sediment sometimes comes from regions of the inlet that are actively scouring because of structural modification or change in flow condition. However, most sediment moving through an inlet is being continually introduced via the longshore drift. Some sediment swept into the inlet deposits on the flood shoal, some deposits in the dredged navigation channel, and some moves out of the inlet to the ebb shoal. Therefore, if a sediment deposition problem such as channel infilling is to be studied in a movable-bed model, it is necessary to introduce the correct quantity of sediment into the model at the appropriate locations.

The best modeling practice is to validate the movable-bed model by reproducing the existing condition before exploring design alternatives. This can only be done for regions of inlets that are close to equilibrium. Provided the movable-bed model bed evolution is in reasonable agreement with the prototype, then the effects of inlet modifications can be studied with greater confidence that the model results are good predictors of the expected behavior of the real inlet. Hughes and Schwichtenberg (1998, 1999) successfully validated a movable-bed model of scour that occurred on the lee side of the Ventura Harbor breakwater. Once validated, the model was used to design suitable scour protection for the breakwater toe.

Example Application: Determine the parameters for a movable-bed model of the entrance channel at Shinnecock Inlet, New York. The size of the model facility requires the prototype-to-model scale ratio to be $N_L = 75$, and the model sediment is quartz sand with a median grain size of $(d_e)_{\mathrm{m}} = 0.13$ mm.

Prototype Parameters:

Inlet width: W_p	$= 245$ m
Maximum depth: $(h_{max})_p$	≈ 14 m
Median grain size: $(d_e)_p$	$= 0.61$ mm (quartz)
Maximum velocity: $\overline{V}_p$	≈ 1.8 m/s (depth-averaged)
Maximum discharge: $(Q_{max})_p$	$\approx 2{,}400$ m^3/s

The inlet width and maximum depth are geometric parameters that scale directly with the length scale, e.g.,:

$$\frac{W_p}{W_m} = N_L = 75 \quad \text{or} \quad W_m = \frac{W_p}{75} = \frac{245\,\text{m}}{75} = \underline{3.3\,\text{m}}(10.7\,\text{ft})$$

The grain-size diameter scale ratio is given as:

$$N_{d_e} = \frac{(d_e)_p}{(d_e)_m} = \frac{0.61\,\text{mm}}{0.13\,\text{mm}} = 4.7$$

which is needed to determine the required velocity scale from Equation 2, i.e.,:

$$N_{\bar{v}} = N_{d_e}^{3/8}\, N_L^{1/8} = (4.7)^{3/8}\,(75)^{1/8} = \underline{3.07}$$

The maximum model velocity comes directly from the velocity scale as:

$$\frac{\overline{V}_p}{\overline{V}_m} = 3.07 \quad \text{or} \quad \overline{V}_m \approx \frac{\overline{V}_p}{3.07} \approx \frac{1.8\,\text{m/s}}{3.07} \approx \underline{0.59\,\text{m/s}}\,(1.9\,\text{ft/s})$$

Recognizing that the discharge scale is simply the velocity scale multiplied by the area scale, i.e.,:

$$N_Q = N_{\overline{V}}\, N_A = N_{\overline{V}}\, N_L^2 = 3.07(75)^2 = \underline{17{,}270}$$

the maximum model discharge is obtained as:

$$\frac{Q_p}{Q_m} = 17{,}270 \quad \text{or} \quad Q_m \approx \frac{Q_p}{17{,}270} \approx \frac{2{,}400\,\text{m}^3/\text{s}}{17{,}270} \approx \underline{0.14\,\text{m}^3/\text{s}}\left(4.9\,\text{ft}^3/\text{s}\right)$$

In summary, the movable-bed model of Shinnecock Inlet should have the following characteristics. These are reasonable parameters well within laboratory capabilities.

Model Parameters

Inlet width: W_m	= 3.3 m
Maximum depth $(h_{max})_m$	≈0.19 m
Median grain size: $(d_e)_m$	= 0.13 mm (quartz)
Maximum velocity: V_m	≈0.59 m/s (depth averaged)
Maximum discharge $(Q_{max})_m$	≈0.14 m^3/s

NOTE: If model velocities are scaled according to Froude scale, the required model velocity would be:

$$\overline{V}_m = \frac{\overline{V_p}}{\sqrt{N_L}} = \frac{1.8\ m/s}{\sqrt{75}} = 0.21 \text{m/s}$$

which is beneath the critical velocity necessary to initiate movement of 0.13 mm quartz sand.

Additional Information. Questions about this note can be addressed to Ms. Jackie Pettway, Jackie.S.Pettway@usace.army.mil This note was produced under the Coastal Inlets Research Program (CIRP). For further information on the CIRP, please contact the CIRP Technical Leader, Dr. Julie Rosati, Julie.Rosati@usace.army.mil . This note should be cited as follows:

Hughes, S. A. (2000). "Simulation of current-induced scour in movable-bed inlet models," ERDC/CHL CETN-IV-26, U.S. Army Engineer Research and Development Center, Vicksburg, MS. (*http://chl.wes.army.mil/library/publications/cetn*)

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Appendix I: Derivation of Scaling Relationship

An empirical expression for equilibrium scour depth as a function of sediment size and maximum discharge per unit width was presented in CETN-IV-18 (Hughes 1999). This relationship was formulated as a balance between the shear stress at the bottom and the critical shear stress of the noncohesive sediment. An unknown coefficient in the theoretical formulation was found using maximum discharge measurements from two inlets. The resulting formula for equilibrium scour depth is restricted to those portions of the inlet channel where currents alone are responsible for movement of sediment by bed load. The equilibrium scour depth relationship is given in terms of discharge per unit length as:

$$h_e = \frac{0.234\, q_e^{8/9}}{\left[g\left(S_s - 1\right)\right]^{4/9} d_e^{1/3}} \tag{10}$$

or in terms of velocity as:

$$\overline{V} = 5.124 \left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8}\, h_e^{1/8} \tag{11}$$

where

h_e = water depth at equilibrium
q_e = maximum equilibrium discharge per unit width
g = gravitational acceleration
S_s = sediment specific gravity [$=\rho_s / \rho_w$]
ρ_s = mass density of sediment
ρ_w = mass density of water
d_e = sand median grain size
$\overline{V}$ = depth-averaged velocity

Similitude of equilibrium scour depths is achieved by assuring that the relationships given by Equations 10 and 11 have the same effect in the movable-bed model as in the real world. The necessary scaling relationship is found by first rearranging Equation 11 into the form of a nondimensional number, i.e.:

$$\frac{\overline{V}}{\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8}\, h_e^{1/8}} = 5.124 \tag{12}$$

Similitude requires maintaining the same value of the nondimensional number in both the model and prototype, or:

$$\left(\frac{\overline{V}}{\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8}\, h_e^{1/8}}\right)_p = \left(\frac{\overline{V}}{\left[g\left(S_s - 1\right)\right]^{1/2} d_e^{3/8}\, h_e^{1/8}}\right)_m \tag{13}$$

where the subscripts *p* and *m* refer to prototype and model, respectively.

Equation 13 can be rearranged into ratios of prototype values of a variable over the corresponding model value. These are termed "scale factors," and they are denoted by an upper case *N*. For example, the scale factor for depth-averaged velocity is:

$$N_{\overline{V}} = \frac{\overline{V}_p}{\overline{V}_m}$$

Replacing all the prototype-to-model ratios with scale factors, and recognizing that gravity will be the same in the model as in the prototype, results in the following scaling relationship:

$$\frac{N_{\overline{V}}}{\left[N_{(S_s-1)}\right]^{1/2} N_{d_e}^{3/8} N_L^{1/8}} = 1 \tag{14}$$

or

$$N_{\overline{V}} = \left[N_{(S_s-1)}\right]^{1/2} N_{d_e}^{3/8} N_L^{1/8} \tag{15}$$

with $N_L = (h_e)_p / (h_e)_m$ representing the length scale of the model. If the model sediment has the same density as the prototype sediment, then $N_{(Ss-1)} = 1$, resulting in a simplified scaling relationship of:

$$N_{\overline{V}} = N_{d_e}^{3/8} N_L^{1/8} \tag{16}$$